Stability of Control Systems - GATE Study Material in PDF

Now that we are done with the Basics of Control Systems and Signal Flow Graph and Block Diagram Reduction Techniques, let us get to the next chapter – **Stability of Control Systems**. In these **free GATE Notes**, we will deal with the **Concept of Stability for Linear Time Invariant (LTI) Systems**. We discuss the conditions that should be satisfied for a LTIsystem to be stable. This article deals with the Stability of Control Systems and covers concepts like **BIBO stability, Condition for Stability, Marginal Stability, Asymptotic Stability, Conditional Stability, Absolute Stability** and **Relative Stability**

This **GATE Study Material** can be **downloaded in PDF** for revision and reference later. These GATE notes are useful for **GATE 2017, BSNL, BARC, DRDO, IES** and other exams.

Recommended Reading –

Laplace Transforms

Control Systems Sensitivity

Block Diagram Algebra

Signal Flow Graph

Concept of Stability for Control Systems

An LTI system is said to be stable if the below two conditions are being satisfied.

1. For bounded input, the system produces bounded output.

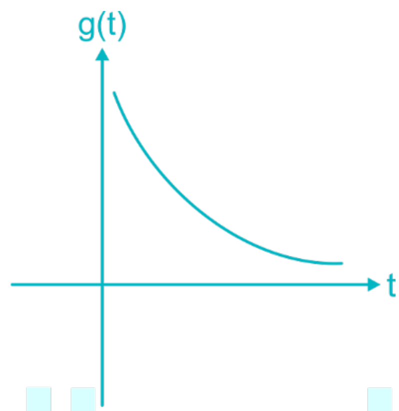
2. In the absence of input, the output tends towards zero irrespective of initial conditions.This phenomenon is also known as **asymptotic stability**.

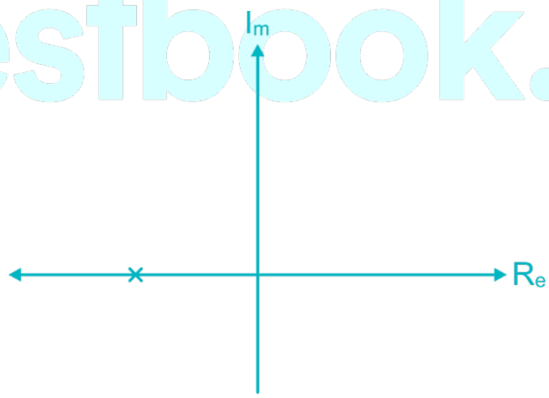
It can also be said as for input r(t) being $|r(t)| \le M_1 < \infty$; the output c(t) will be $|c(t)| \le M_2 < \infty$.

$$\text{If } C(s) = L\big(r(t)\big), R(s) = L\big(r(t)\big) \text{ and } \frac{C(s)}{R(s)} = G(s),$$

$g(t)=L^{-1}(G(s))$is the response for given input. The nature of this response can be gaugedfrom the poles of G(s) which are also known as the roots of the characteristic equation.

$$\text{For instance, if } G(s) = \frac{1}{s+1}, \text{then } g(t) = e^{-t}. \text{ If we plot g(t) against t}$$





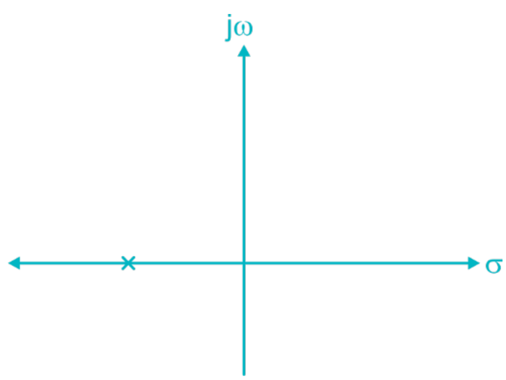
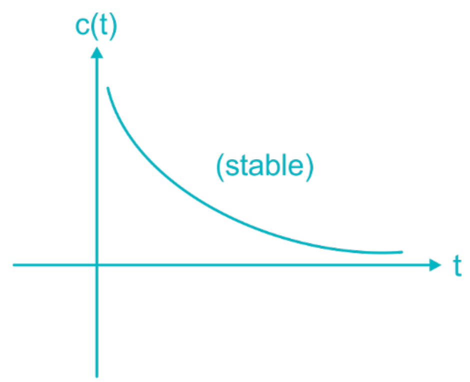
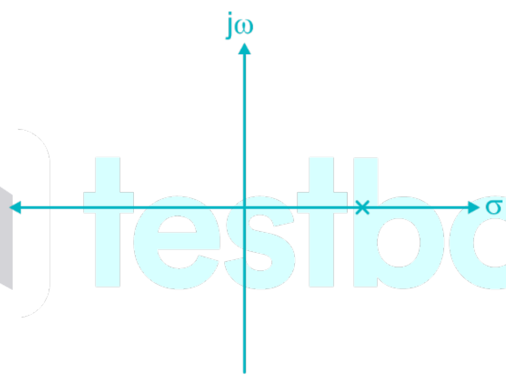
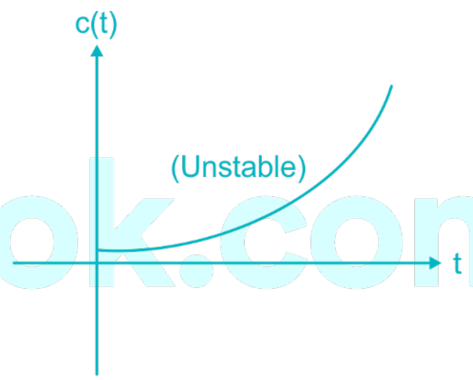
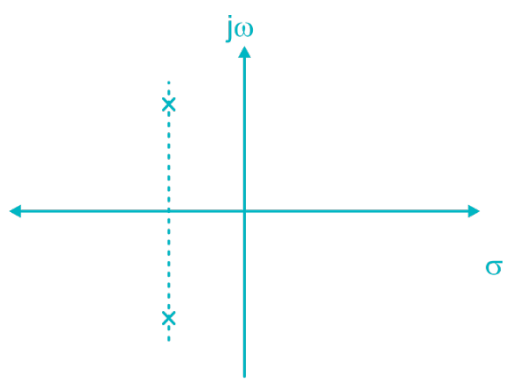
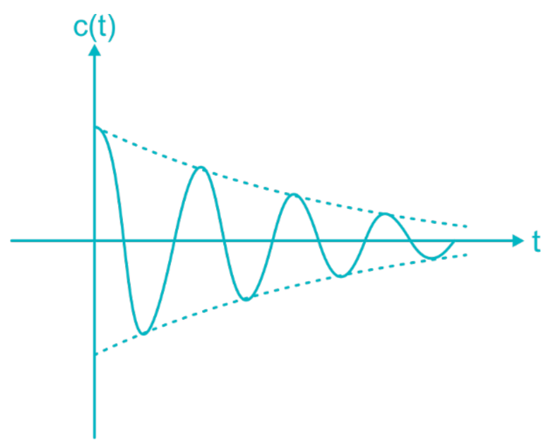
Roots is S - phere

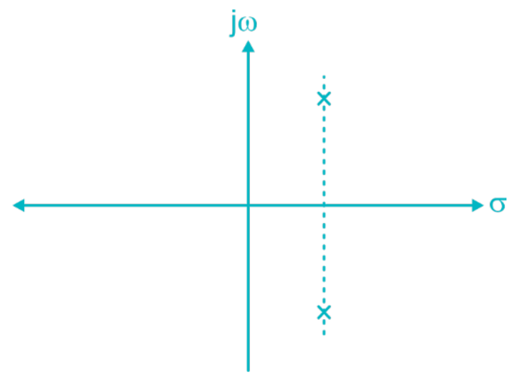
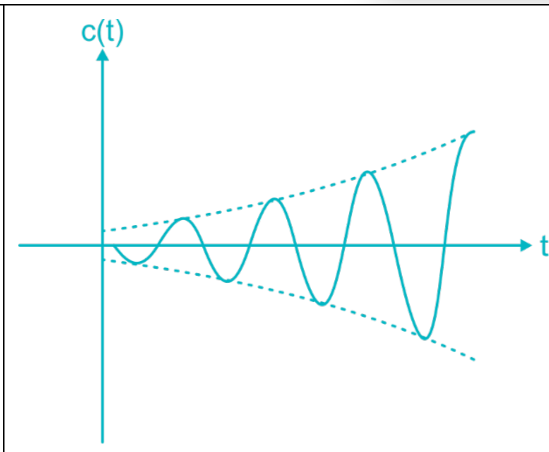
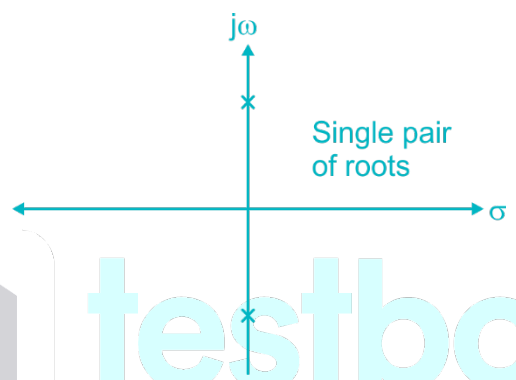
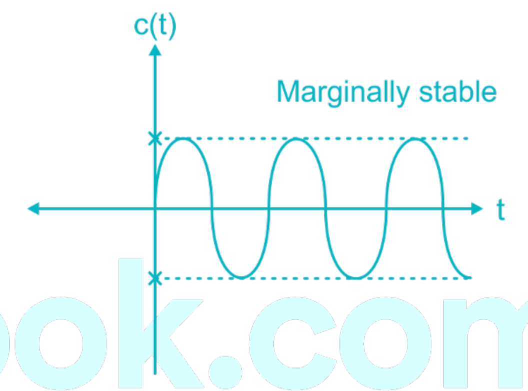
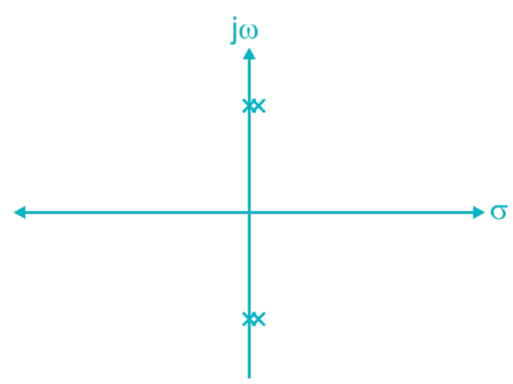
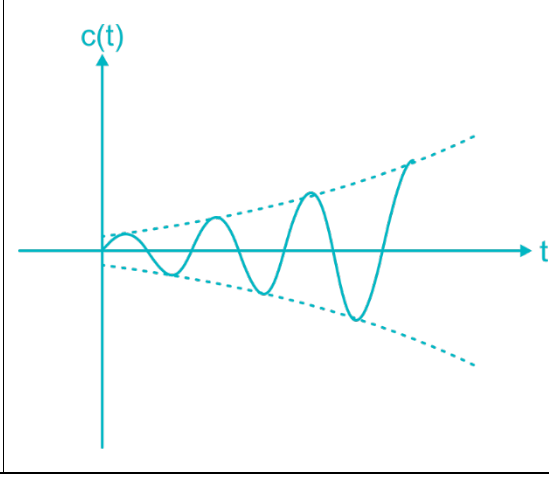
As seen above, a pole in negative region leads to decaying response as shown. Due to this,the system in question will be stable as per the definition of stability mentioned previously.

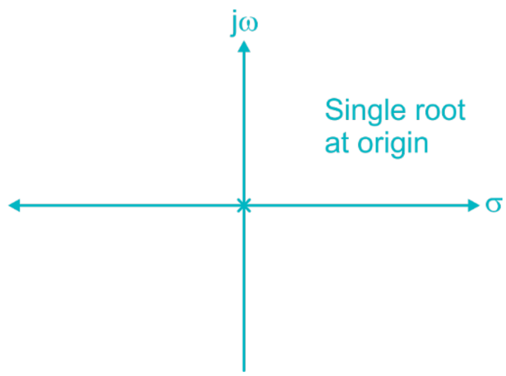
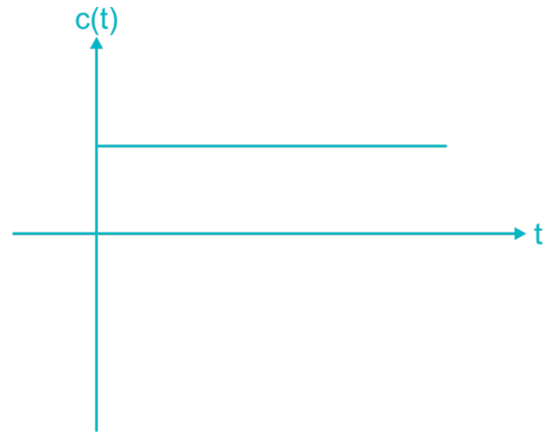
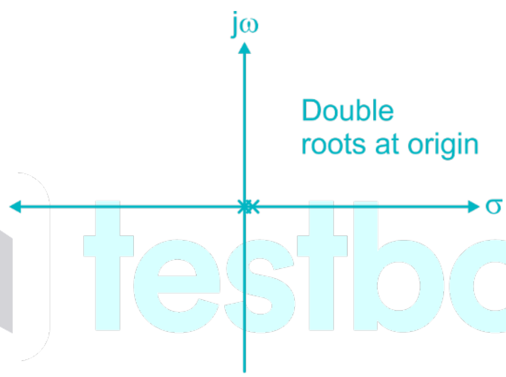
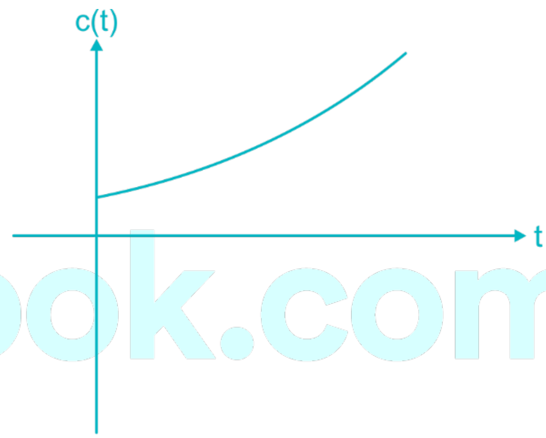
Similarly, for $G(s) = \frac{1}{s-1}$ having root in positive region, $g(t) = e^{t}$ which is increasingexponential. This response is unbounded and hence the system in this case is **unstable**.

If r(t)is Impulse, then R(s)=1, C(s)=G(s)can also be termed as **impulse response**.

Now, the impulse response of system with respect to position of poles can be illustrated as('X' indicating as position of poles)

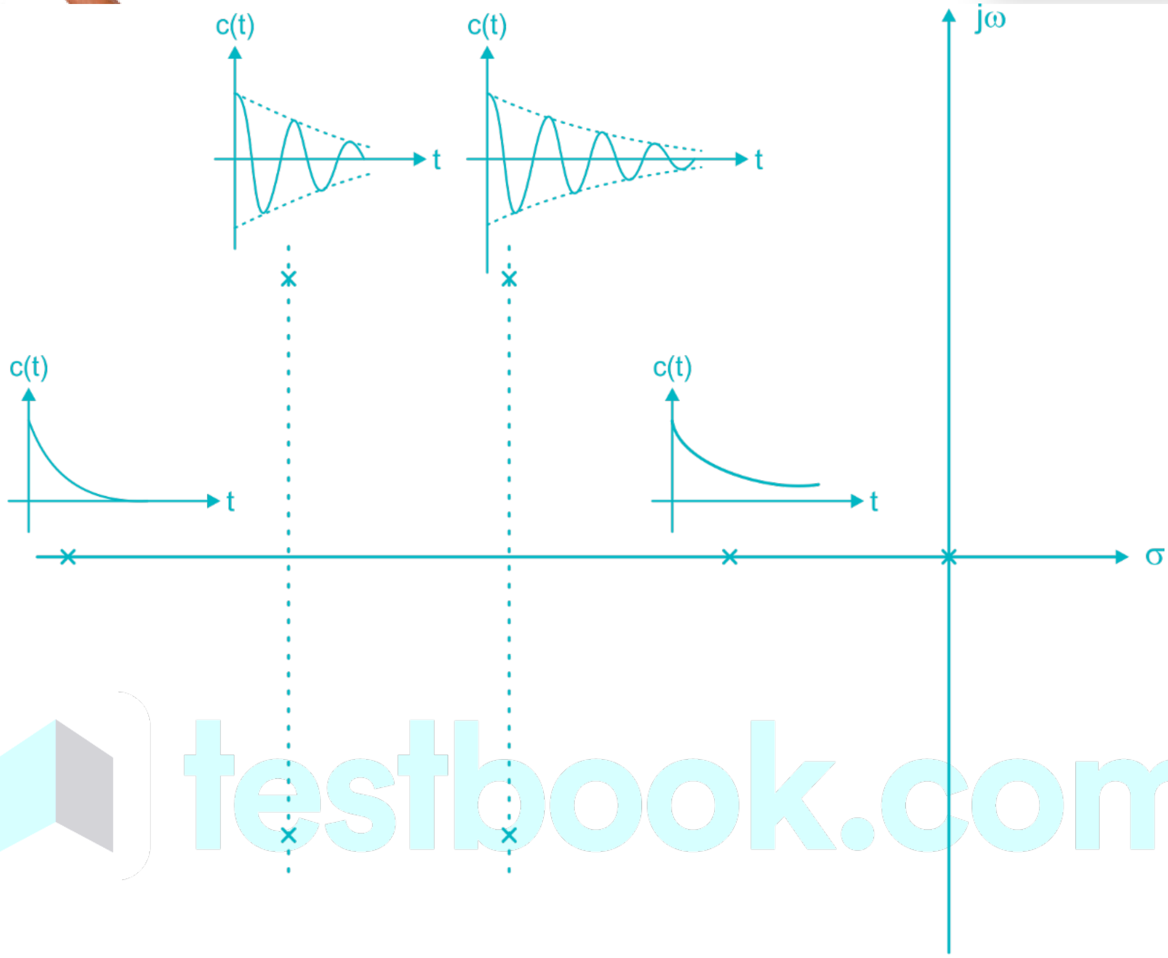
I)		
II)		
III)		

IV)	jω σ	c(t) t
V)	jω Single pair of roots σ	c(t) Marginally stable t
VI)	jω σ	c(t) t

VII)		
VIII)		

For a single pair of roots on imaginary line, the system is **Marginally Stable**, which means a slight disturbance can push it towards instability. For double pair of roots at imaginary line, the system will be unstable. Similar case will be for a single root at origin and a pair of roots at origin. As shown above, for the roots which are in negative s-plane i.e. having negative real part the system will be stable. On the other hand, roots having positive real part constitute for an unstable system.

Another property of system which we look at is **Relative Stability**. It is the measure of relative response of transients dying with respect to time for different poles. The farther the poles are on left side of negative S – plane, the more quickly the transients will dry. The relative response of system with respect to position of poles is illustrated below.



This was the basic concept of Stability of Control Systems. In next article we will explore the Routh-Hurwitz Stability Criteria.